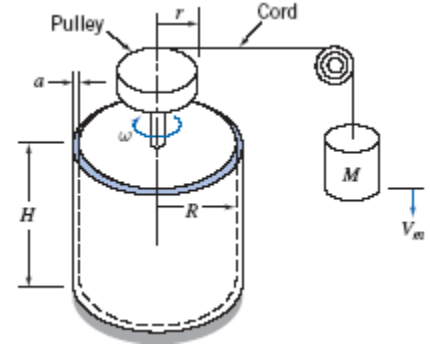


Problem 2.61

[Difficulty: 3]

2.61 The viscometer of Problem 2.60 is being used to verify that the viscosity of a particular fluid is $\mu = 0.1 \text{ N} \cdot \text{s}/\text{m}^2$. Unfortunately the cord snaps during the experiment. How long will it take the cylinder to lose 99% of its speed? The moment of inertia of the cylinder/pulley system is $0.0273 \text{ kg} \cdot \text{m}^2$.



Given: Data on the viscometer

Find: Time for viscometer to lose 99% of speed

Solution:

The given data is $R = 50 \cdot \text{mm}$ $H = 80 \cdot \text{mm}$ $a = 0.20 \cdot \text{mm}$ $I = 0.0273 \cdot \text{kg} \cdot \text{m}^2$ $\mu = 0.1 \cdot \frac{\text{N} \cdot \text{s}}{\text{m}^2}$

The equation of motion for the slowing viscometer is $I \cdot \alpha = \text{Torque} = -\tau \cdot A \cdot R$

where α is the angular acceleration and τ is the viscous stress, and A is the surface area of the viscometer

The stress is given by $\tau = \mu \cdot \frac{du}{dy} = \mu \cdot \frac{V - 0}{a} = \frac{\mu \cdot V}{a} = \frac{\mu \cdot R \cdot \omega}{a}$

where V and ω are the instantaneous linear and angular velocities.

Hence $I \cdot \alpha = I \cdot \frac{d\omega}{dt} = -\frac{\mu \cdot R \cdot \omega}{a} \cdot A \cdot R = -\frac{\mu \cdot R^2 \cdot A}{a} \cdot \omega$

Separating variables $\frac{d\omega}{\omega} = -\frac{\mu \cdot R^2 \cdot A}{a \cdot I} \cdot dt$

Integrating and using IC $\omega = \omega_0$ $\omega(t) = \omega_0 \cdot e^{-\frac{\mu \cdot R^2 \cdot A}{a \cdot I} \cdot t}$

The time to slow down by 99% is obtained from solving $0.01 \cdot \omega_0 = \omega_0 \cdot e^{-\frac{\mu \cdot R^2 \cdot A}{a \cdot I} \cdot t}$ so $t = -\frac{a \cdot I}{\mu \cdot R^2 \cdot A} \cdot \ln(0.01)$

Note that $A = 2 \cdot \pi \cdot R \cdot H$ so $t = -\frac{a \cdot I}{2 \cdot \pi \cdot \mu \cdot R^3 \cdot H} \cdot \ln(0.01)$

$$t = -\frac{0.0002 \cdot \text{m} \cdot 0.0273 \cdot \text{kg} \cdot \text{m}^2}{2 \cdot \pi} \cdot \frac{\text{m}^2}{0.1 \cdot \text{N} \cdot \text{s}} \cdot \frac{1}{(0.05 \cdot \text{m})^3} \cdot \frac{1}{0.08 \cdot \text{m}} \cdot \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} \cdot \ln(0.01) \quad t = 4.00 \text{ s}$$